Algebraic Combinatorics School 2015

Place: KIAS 1114, 1503

Date: 10 Feb. ~ 13 Feb., 2015

KIAS | 수학난제연구센터
Center for Mathematical Challenges

1 Schedule

	10 Feb.	11 Feb.	12 Feb.	13 Feb.
08:30~09:30		Breakfast	Breakfast	Breakfast
09:30~10:30		Lecture K-2	Lecture K-3	Lecture K-4
11:00~12:00		Lecture C-2	Lecture C-3	Lecture C-4
12:00~14:00		Lunch	Lunch	Lunch
14:00~15:00	Lecture K-1	Lecture L-2	Lecture L-3	Lecture L-4
15:30~16:30	Lecture C-1	Discussion 1	Discussion 3	Discussion 5
17:00~18:00	Lecture L-1	Discussion 2	Discussion 4	
18:00~	Dinner	Banquet	Dinner	

K: Jae-Hoon Kwon/ C: Soojin Cho/ L: Kyungyong Lee

2	Lecture K

Speaker	Jae-Hoon Kwon (Sungkyunkwan University)
Title	Introduction to symmetric functions and representation theory
Abstract	In this lecture, we give an introduction to the theory of symmetric functions in connection with representations of symmetric groups and general linear groups. It is intended for those who are interested in these topics but not yet familiar with them. The lecture is divided into the following: 1.Symmetric functions 2.Combinatorics of Young tableaux 3.Representations of symmetric groups 4.Representations of general linear groups These topics will (hopefully) cover very classical results on combinatorial representation theory and algebraic combinatorics, which are necessary background to understand modern research topics like representations of quantum groups and Hecke algebras, categorification and so on.
References	1.I. G. Macdonald, Symmetric functions and Hall polynomials, Oxford University Press, 2nd ed., 1995. (Chapter 1) 2.W. Fulton, Young tableaux, London Mathematical Society Student Texts, 35. Cambridge University Press, 1997.(Parts I and II) 3.R. Stanley, Enumerative Combinatorics, Vol 2, Cambridge University Press, 1999. (Chapter 7 and Appendix A)

Lecture 1. Symmetric functions

We introduce the notion of symmetric functions and review some of its basic properties. We study linear bases of the ring of symmetric functions including a basis of Schur functions, the Hall inner product, and its Hopf algebra structure.

Lecture 2. Combinatorics of Young tableaux

We introduce a combinatorial algorithm for Young tableaux, which is called the Schensted's bumping. Based on this, we study the celebrated Robinson-Schensted-Knuth (simply RSK) correspondence and its various applications. We also study the Littlewood-Richardson coefficients, which give multiplicative structure constants for the ring of symmetric functions with respect to Schur functions.

Lecture 3. Representations of symmetric groups

We briefly review some basic background on representation theory of finite groups. We then explain how the representation theory of symmetric groups over complex numbers is related with the theory of symmetric functions. More precisely, we will see that the character ring of symmetric groups is isomorphic to the ring of symmetric functions and in particular the irreducible characters of symmetric groups are completely determined by a transition matrix between two linear bases of the ring of symmetric functions.

Lecture 4. Representations of general linear groups

We review well-known results on complex finite-dimensional representations of general linear groups, and then explain its connection with the theory of symmetric functions. We give representation theoretic interpretations or applications to representation theory of combinatorial results in Lectures 1 and 2. If time permits, we explain two important duality theorems: Schur-Weyl duality, which connects the representations of symmetric groups and general linear groups, and Howe duality, which underlies the RSK correspondence.

3	Lecture C

Speaker	Soojin Cho (Ajou University)
Title	Combinatorics of Coxeter Groups; a basic
Abstract	Coxeter group을 정의하고, 다음 관련주제에 관한 기본이론을 A type을 중심으로 소개한다. ■ Bruhat order ■ Reduced decompositions ■ Stanley Symmetric functions ■ Quasi-symmetric functions
References	1.Björner, Anders; Brenti, Francesco “Combinatorics of Coxeter groups.” Graduate Texts in Mathematics, 231. Springer, New York, 2005 2.Sagan, Bruce E. “The symmetric group. Representations, combinatorial algorithms, and symmetric functions.” Second edition. Graduate Texts in Mathematics, 203. Springer-Verlag, New York, 2001 3.Stanley, Richard P. “Enumerative combinatorics.” Vol. 2. Cambridge Studies in Advanced Mathematics, 62. Cambridge University Press, Cambridge, 1999.

Lecture 1. Coxeter groups and Bruhat order

We first introduce the 'Coxeter systems' and nice properties that they have; exchange property and deletion property. Then, we consider a poset structure on Coxeter groups, called (weak) Bruhat order with some combinatorial examples at least for type A_n.

Lecture 2 and 3. Quasi-symmetric functions and Stanley Symmetric functions

The notion of 'quasi-symmetric functions' will be introduced. Then we will see how Schur functions are written as sums of 'fundamental' quasi-symmetric functions, in which the statistics 'descent' on the symmetric group (Coxeter group of type A) plays a role.

Stanley symmetric functions were defined to enumerate the reduced expressions of elements in Coxeter groups: We give the definition of Stanley symmetric functions in terms of quasi symmetric functions, and see how the theory of symmetric functions is used to count the number of reduced expressions. Edelman-Greene correspondence which combinatorially shows that Stanley symmetric functions are positive sums of Schur functions will also be mentioned. We consider Coxeter groups of other types in this context also, if time permits.

Lecture 4. Poincaré series and Eulerian polynomials

We look at generating functions of statistics 'length', 'descent', and both on Coxeter groups. Recursion formulae and some nice properties they satisfy will be discussed.

참고문헌:

1. Björner, Anders; Brenti, Francesco "Combinatorics of Coxeter groups." Graduate Texts in Mathematics, 231. Springer, New York, 2005

2. Stanley, Richard P. "Enumerative combinatorics." Vol. 2. Cambridge Studies in Advanced Mathematics, 62. Cambridge University Press, Cambridge, 1999.

3. Sagan, Bruce E. "The symmetric group. Representations, combinatorial algorithms, and symmetric functions." Second edition. Graduate Texts in Mathematics, 203. Springer-Verlag, New York, 2001

4	Lecture L

Speaker	Kyungyong Lee (Wayne State University and KIAS CMC)
Title	Cluster algebras and related combinatorics
Abstract	Cluster algebras were discovered by Fomin and Zelevinsky in 2001. Since then, they are shown to be related to many branches of mathematics and physics. With very active development in the last decade, they become fundamental objects. A cluster algebra is a commutative algebra with distinguished generators called cluster variables. These cluster variables are Laurent polynomials obtained from highly nontrivial recursive relations. It is a very important problem to find combinatorial formulas for these Laurent expressions. These formulas will lead to lots of applications in algebra, geometry, topology, analysis and physics as well as combinatorics. We explain some known combinatorial formulas for certain special cases including the rank 2 case and the ones coming from Riemann surfaces. Along the way, we explore a number of interesting combinatorial objects : snake graphs, perfect matchings, Dyck paths and compatible pairs. We focus on very explicit computations for such objects.

Lecture 1. Introduction to cluster algebras

A cluster algebra is a commutative algebra with distinguished generators called cluster variables. These cluster variables are constructed from certain recursive relations. To each directed graph (quiver), we define the associated cluster algebra. We also prove that the cluster variables are Laurent polynomials.

Lecture 2. Cluster algebras coming from discs

We explain how cluster algebras arise from triangulations of discs with marked points on the boundary. In this case we give combinatorial formulas for the Laurent expressions of cluster variables in terms of T-paths and globally compatible collections on Dyck paths.

Lecture 3. Cluster algebras coming from Riemann surfaces

We explain how cluster algebras arise from triangulations of Riemann surfaces with marked points and (possibly empty) boundaries. In this case we give combinatorial formulas for the Laurent expressions of cluster variables in terms of snake graphs and perfect matchings.

Lecture 4. Cluster algebras of rank 2

We define noncommutative cluster algebras of rank 2, using Kontsevich automorphisms. In this case we give a combinatorial formula for cluster variables in terms of Dyck paths and compatible pairs.